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When Am I Ever Going to Use This in the Real World? Cognitive Flexibility and Urban Adolescents' Negotiation of the Value of Mathematics

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Many adolescent learners have difficulty understanding the relevance of mathematics for their lives. This problem is particularly pernicious among Black and Latino adolescents who often face cultural stigma that can affect their perceived value of mathematics. The present study used concurrent nested mixed methods to explore this issue in 419 urban Black and Latino adolescents. Structured classroom observations, a computerized cognitive assessment, and surveys were used to examine how teacher math applications (TMAs) and adolescent cognitive flexibility interact to predict students' valuing of mathematics. From a subset of the larger sample ($n = 37$), semistructured qualitative interviews were used to understand how these adolescents came to view mathematics as a transformative tool in their lives, particularly in the face of cultural stigma. The quantitative results revealed that TMAs were associated with students' value of mathematics. However, these results also illustrated how TMAs interacted with adolescent cognitive flexibility to predict students' growth in valuing mathematics over the school year. The qualitative interviews corroborated the quantitative findings, but also revealed 3 themes that extended the quantitative results, uncovering racialized facets of valuing mathematics. The 3 themes that emerged were: *utility orientations*, *alternative messengers*, and *resisting stigma and protecting collective identity*. Altogether, these results demonstrated the role real-world applications, race, and adolescent cognition can have in urban mathematics classrooms. These findings suggest teachers' sensitivity to these issues can support Black and Latino adolescents' persistence in mathematics and understanding of self.

Educational Impact and Implications Statement

This study suggests that teacher messages about the real-world relevance of mathematics matters in shaping how urban Black and Latino adolescents value mathematics. However, this study showed math teachers infrequently connected their instruction to the real world. To support adolescents' value of mathematics, teachers need to better understand how to use real-world applications in the classroom that are sensitive to race, context, and adolescent cognition.

Keywords: mathematics identity, cognitive development, concurrent mixed methods, reform mathematics, urban education

"When am I ever going to use this in the real world?" A few years ago, I sat in the back of an algebra one class working on linear equations when one precocious student slowly raised his hand and posed this question to his teacher. With only a couple minutes left in the period, the teacher decided to entertain the question, diving into somewhat of an ill-prepared explanation on the real-world importance of linear equations. However, the student's blank gaze in return revealed that, despite the teacher's best intent, this explanation went completely over his head. A quick glance around the room showed several other students were also confused. After a few seconds of awkward silence and stares around the classroom, the teacher hastily tried to redirect the conversation, reminding the class of the take-home assignment due at the end of the week. Within the next minute, the bell rang and the students exited for their next class.

This episode, although not rare within many secondary mathematics classrooms, underscores the complexity of connecting formal mathematics to adolescents' lives. Making these connections is deceptively difficult for both students and teachers to do (Bransford, Brown, & Cocking, 1999) and multiple factors at the classroom, teacher, and student level make it challenging for adoles-

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cents to realize the value of mathematics for their lives. Moreover, Black and Latino populations are largely underrepresented in math-related majors and careers. Recent work has linked this underrepresentation to math value beliefs that develop during the middle and high school years (Maltese & Tai, 2011; McCoy, 2005; Wang & Degol, 2013). Thus, supporting Black and Latino adolescents in understanding the value of mathematics during these precollege years can be instrumental in creating opportunities for access and equity in higher education and beyond.

The present study investigated the role of *teacher math applications* (TMAs; i.e., teacher applications that connect classroom math to the outside world) and *adolescent cognitive flexibility* (i.e., mental adaptability to be able to think across multiple dimensions) in predicting adolescents' growth in valuing mathematics. These issues are examined against the backdrop of cultural stigma and systemic inequities that many urban Black and Latino adolescents face, which can create additional obstacles toward learning and valuing mathematics (Martin, 2012). Using classroom observations, a computerized cognitive assessment, and surveys, I assessed the interaction between TMAs and adolescent cognitive flexibility on students' growth in valuing mathematics. Further, using semistructured qualitative interviews, I probed students' classroom experiences, received TMAs, and math value beliefs to understand the adolescents' perspectives through their own words and experiences. This collection of methods allowed for objective analysis of how teacher and student factors interacted to predict net growth in valuing mathematics while simultaneously employing a holistic approach to understand the adolescents' perspectives in context.

Literature Review

Valuing Mathematics During Adolescence

Mathematics continues to be a subject area of disdain for many adolescents (Hersh & John-Steiner, 2011), with research noting declines in adolescents' math value and efficacy during middle and high school (Fredricks & Eccles, 2002; Frenzel, Goetz, Pekrun, & Watt, 2010; McCoy, 2005; Watt, 2004). Even compared with the other STE domains (i.e., science, technology, and engineering), mathematics has consistently received the lowest interest ratings by adolescents (Munce et al., 2012), primarily due to their perceptions of its ever-increasing complexity, abstract density, and the amount of effort needed to succeed (Hidi, 2000).

Although declines in math value also exist for Black and Latino adolescents, systemic inequities and cultural stigma can further impinge their opportunities to engage in mathematics and internalize the value of it (Martin, 2012). For example, Black and Latino students in urban schools are more likely to have less experienced teachers (Borman & Dowling, 2008) and less likely to take algebra one before ninth grade (Nord et al., 2011). Beyond these opportunity gaps (Milner, 2012), Black and Latino adolescents also face stigma, such as perceptions of intellectual inferiority by their mathematics teachers (Spencer, 2009) that can present unique psychological hurdles (e.g., low value, lack of belonging, low efficacy). Thus, in this study, I examine value of mathematics as a central motivational construct related to promise and persistence for students who have been historically underserved in mathematics classrooms.

Attainment value is the importance students attach to a domain because they view it as self-defining or a reflection of their identity (e.g., I see myself as a math person; Wigfield, 1994). Attainment value, in combination with an expectation for success, is a vital ingredient to persistence, effort, and goal setting in mathematics (Wang, Degol, & Ye, 2015). Research has shown that when students were able to perceive the value of mathematics, they were more likely to engage in learning and develop school and career aspirations involving mathematics (Durik, Vida, & Eccles, 2006; Simpkins, Davis-Kean, & Eccles, 2006).

In motivation research, there exist other value constructs that are related yet distinct from attainment value (Wigfield & Cambria, 2010). A few of these include *intrinsic value* (i.e., natural enjoyment or interest in a subject), *utility value* (i.e., perceived usefulness of a subject; Wigfield & Eccles, 1992), and *abstract value* (i.e., perceived global value of a domain, without importing specific value for the individual self; Mickelson, 1990). Mickelson (1990) found abstract values to be popular held beliefs among urban Black adolescents. She attributed these findings to a tension Black adolescents experience between structural oppression, that limits the potential education can have for their lives, and the perceived benefits education has for other, socially advantaged, children.

Considering attainment and other achievement values within the urban school context, when Black and Latino adolescents ask, "When am *'I'* ever going to use this in the real world?" this may be more than just questioning the usefulness of mathematics (i.e., utility value) that many adolescents often wonder. Rather, this question likely emphasizes the '*I*' here; in other words, the agency in one's mathematics identity in the face of social disadvantages. Derived from the math education literature, mathematics identity refers to "deeply held beliefs that individuals develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives" (Martin, 2012, p. 57). Thus, urban Black and Latino adolescents may seek to understand how mathematics can inform who they are or who they can become beyond the utility of helping them in a future class or career. They may seek to understand mathematics to help navigate their immediate environment, especially their perceived social challenges and stigmata they face in school and society.

Teacher Mathematics Applications

All of the aforementioned issues have led mathematics educators to reform traditional mathematics education practices to accommodate the learning needs of an increasingly diverse student population nationwide (National Council of Teachers of Mathematics, 2000). The mathematics reform movement has encouraged teachers to move beyond simply promoting rote procedural competencies and abstract reasoning, and progress toward developing students' critical thinking and mathematical identities (Berry, 2003; Schoenfeld, 2014). This requires mathematics teachers to make connections between procedures, concepts, and contexts that enhance critical thinking opportunities and allow students to see themselves in the mathematics (Gutstein, 2007). Teachers can facilitate these connections through integrating TMAs into their instruction.

TMAs are opportunities teachers create to allow students to: (a) see mathematics as a web of interconnected concepts, versus a collection of disconnected rules or procedures; and (b) connect formal mathematics with experiences outside of the classroom. This conceptualization was based on prior research regarding high leverage practices in mathematics instruction as well as national standards (Ottmar, Rimm-Kaufman, Larsen, & Berry, 2015; National Council of Teachers of Mathematics, 2000). TMAs can unfold through a variety of instructional choices, such as word problems that integrate students' experiences (e.g., "Tops are 15% off and jeans 20% off at your favorite store . . ."), relevant teacher analogies (e.g., relating percentages to pizza pies), discussions of how math is used in contemporary society (e.g., predicting presidential or local elections), analysis of real data (e.g., collecting classmates' data to see the relation between height and shoe size), tangible math representations (e.g., algebra tiles), and mathematical modeling of real world phenomena (e.g., writing an equation to represent the relation between years of education and money earned; Gainsburg, 2008).

Despite the reform efforts, research shows that mathematics teachers in the United States have made little progress in using real world applications in their instruction (Gainsburg, 2008). When American mathematics teachers do include TMAs, it is generally cursory and infrequent (i.e., once per week or less) compared with Eastern countries (e.g., Japan) where teachers spend half their instructional time implementing TMAs (Hiebert et al., 2008). Further, when American teachers integrate TMAs into their instruction, research has shown they do so to reinforce abstract content knowledge and rarely to promote student discovery, critical thinking, or mathematical identity (Gainsburg, 2008).

Some teachers may also believe that providing TMAs is only appropriate for advanced students (Nathan & Koedinger, 2000), as they have demonstrated mastery over the pure mathematics and are more likely to understand the applied connections. However, this perspective can perpetuate educational inequity for struggling or historically underserved students in mathematics, such as urban Black and Latino adolescents. Research has uncovered how understanding mathematics in and through everyday experiences developed students' conceptual understanding, particularly for those students struggling with the content (Greeno & Middle-School Mathematics Through Applications Project Team, 1997; Walkington, 2013). Furthermore, posing authentic problems related to students' real world cultural experiences increased student engagement and shifted students' beliefs about mathematics so that they saw it as a transformative tool in their lives (Turner, Gutiérrez, Simic-Muller, & Díez-Palomar, 2009).

Despite considerable interest on the topic of TMAs, there is little research that studies this from varied angles, using multiple sources of data for triangulation. Understanding how TMAs relate to achievement values (e.g., attainment) among urban Black and Latino adolescents as well as the understudied role adolescent cognitive development plays in how TMAs are processed are important for advancing both research and practice in mathematics education.

Cognitive Flexibility and Making Meaning in Mathematics

Adolescence is a developmental stage marked by identity exploration (Luyckx, Teppers, Klimstra, & Rassart, 2014; Marcia, 1966). This exploration naturally extends into the classroom where students try to make meaning of the subject matter in ways that inform their own identity. Neurological changes in the prefrontal cortex play a central role in adolescents' propensity toward meaning-making and identity exploration by affording them the executive capacity to think abstractly and hypothetically (e.g., to think about possibilities not grounded in concrete reality), think self-reflectively (e.g., metacognition), and think across multiple dimensions simultaneously (Keating, 2004; Kuhn, 2009). These cognitive capabilities develop through the refinement of two prefrontal functions during adolescence: working memory and cognitive flexibility.

Working memory holds a limited amount of information in active consciousness while mentally "working" on that information toward some goal (e.g., mental math; DeStefano & LeFevre, 2004). Cognitive flexibility refers to the mental adaptability and versatility that allows adolescents to think across multiple perspectives or "think outside the box" (Diamond, 2013 p. 152). Thus, cognitive flexibility is likely the primary cognitive function that supports recognition of how learning in one situation applies to a different situation. Despite the benefits of cognitive flexibility, cognitive load theory (Sweller, Van Merriënboer, & Paas, 1998) suggests cognitive flexibility becomes increasingly difficult to do when working memory becomes oversaturated with information. Thus, cognitive flexibility is functionally dependent on working memory capacity. To illustrate what this might mean within the context of secondary mathematics classrooms, working memory capacity may easily become saturated by the multiple rules and steps of a newly learned algebraic procedure. At the same time, adolescents' enhanced desire for meaning-making can lead them to seek information about the relevance of this work for their identity, resulting in additional abstract information for them to process beyond the algebra. This saturation of working memory ultimately inhibits cognitive flexibility, thereby limiting the capacity to perceive the broader importance of the concept outside of the immediate classroom context. However, an explicit application based in students' experiences can actually ease cognitive load by 'grounding' abstract concepts (Goldstone & Son, 2005; Koedinger & Nathan, 2004). In this way, TMAs can support efficient adolescent processing, whereas vague, abstract, or absent applications may only compound cognitive load.

Ultimately, understanding how abstract mathematics is connected to abstract identity may be more cognitively taxing for adolescents to process than understanding the pure mathematics alone. Although cognitive flexibility is related to the ability to understand abstract mathematical concepts (LeFevre, DeStefano, Coleman, & Shanahan, 2005), little research has examined if cognitive flexibility accounts for whether adolescents can effectively process TMAs and internalize these messages for negotiating value of the mathematical content for their lives. This negotiation, though challenging, is important for developing a personal ownership of mathematics that endures over time (i.e., mathematics identity). Supporting urban Black and Latino adolescents through this negotiation process is particularly important, as they contend with social stigmata and broader societal messages that denigrate mathematics as irrelevant and an area of cultural deficiency (Martin, 2012).

The Present Study

This study used a concurrent nested mixed-method design (Creswell, 2003) to examine the relation between TMAs, cognitive flexibility, and value of mathematics among urban Black and Latino adolescents. Five research questions were posed.

Research Question 1: Do TMAs predict growth in students' value of mathematics over the course of one school year?

Research Question 2: Does cognitive flexibility moderate the relation between TMAs and growth in students' value of mathematics?

Research Question 3: Do TMAs predict growth in students' cognitive flexibility over the course of one school year?

To answer all three questions, I controlled for important student and teacher covariates. Based on my review of the literature above, I expected TMAs to predict growth in students' value of mathematics. Additionally, I expected students who received frequent TMAs and had higher cognitive flexibility scores to exhibit more growth in valuing mathematics than students who scored lower on cognitive flexibility. Finally, despite limited previous research regarding the third question, I anticipated classrooms with greater instances of TMAs would predict gains in student cognitive flexibility over the school year.

To address the final two research questions I used qualitative semistructured interviews to probe the adolescents' experiences and beliefs regarding their value of mathematics.

Research Question 4: Do the adolescent interview narratives corroborate the quantitative findings?

Research Question 5: How do adolescents make meaning of TMAs and come to understand mathematics as a transformative tool in their lives?

The interviews facilitated a personalized articulation of the study's main constructs, which allowed for a few important opportunities beyond what the quantitative data could show alone. First, the interviews honored the voices of the adolescents, positioning them as the experts on their own experiences. Further, the interviews also helped reveal instances of consistency and inconsistency between the adolescents' narratives and the quantitative findings. Finally, the interviews helped unearth students' value of mathematics in context, revealing how race, stigma, and sociopolitical histories influenced what mathematics means to these adolescents.

Method

Participants

This study is first year data from an ongoing 5-year longitudinal project that examined motivation in mathematics among urban Black and Latino adolescents during middle and high school. The present data was collected during the 2014–2015 school year in one large city in New Jersey, United States. Fifty-two percent of this city's residents identified as Black or African American and 33.8% as Latino. The median income was $35,659 with approximately 25% of the population living in poverty (U.S. Census Bureau, 2010).

After Institutional Review Board (IRB) approval and school district commitment, students were recruited through in-person announcements across five schools. A purposive nonprobability approach was used to recruit schools that varied by racial composition and academic rigor. Students were asked to return guardian consent forms and sign assent forms to participate. Teachers of these classrooms also gave consent to be observed. The response rate for student and guardian consent was approximately 64%, resulting in a final sample of 419 students across 31 fifth-, sixth-, seventh-, and ninth-grade[1] math classrooms (M age = 12.9 years; 53% female). There was 6.2% student attrition by the end of school year.

Two schools were kindergarten through eighth grade, two were high schools with grades nine through 12, and one was a high school with grades seven through 12. All schools had over 85% of students eligible for free-reduced lunch. Two schools were magnet public schools ranked among the best schools in the state (U.S. News & World Report, 2017). Racially, the first magnet school was 61% Latino and 26% Black, and the second magnet school was 47% Latino and 30% Black (New Jersey Department of Education, 2015). The three remaining schools were approximately 90% Black, with small contingents of Latino subgroups. These schools ranked within the bottom quarter of schools in the state for academic performance (New Jersey Department of Education, 2015). The final sample across all schools was 56% Black, 27% Latino or one of its ethnic subgroups, 3% White, and 13% as other. Overall, the sample mirrored the racial demographics of the larger city and contained considerable diversity with regard to school racial composition, academic rigor, and performance.

Design

This study used a concurrent nested mixed-method design. Because any single method has inherent limitations, multiple methods provide the advantage of producing multiple data sources to address the limitations of each singular method (Creswell, 2003). Concurrent suggests the multiple methods were employed at the same time (vs. in sequence); while nested indicates there was a predominant guiding method, in which the other method was "nested" (Creswell, Plano Clark, Gutmann, & Hanson, 2003). Concurrent nested designs are used to (a) cross-validate or corroborate findings across multiple data sources and (b) allow the nested method to address a nuanced question embedded within the dominant question (Creswell, 2003).

In this research, the quantitative questions guided the overall study on students' value of mathematics through the examination of teacher and student characteristics. The qualitative questions were nested to corroborate the quantitative findings and also explore ways in which students' value of mathematics was racialized and contextualized. The surveys, as part of the quantitative method, were administered twice during the school year, once in early October (i.e., baseline), and again in June (i.e., year-end). The classroom observations were conducted from November

[1] Eighth graders were not sampled during the current year of the longitudinal study due to logistical limitations at two of the schools. These logistical issues were beyond the scope of this study.

through March across the 31 mathematics classrooms. Finally, the semistructured interviews were conducted with a subsample of the larger survey sample from January through June. Thus, the multiple methods were employed concurrently during the school year.

Data Collection

Student questionnaires and cognitive assessments. Student questionnaires and cognitive assessments were completed online in the school computer labs using individual computers. Trained research assistants monitored the survey administration. The questionnaire lasted 30 to 40 min and contained various scales that measured attainment value of mathematics and other classroom and motivational constructs. Directly following the questionnaire, all students played a series of computerized games that measured various cognitive capabilities, including cognitive flexibility. Students were given a small monetary incentive after completing the questionnaire and cognitive assessment.

Attainment value of mathematics. Attainment value of mathematics (hereafter 'attainment value') was measured through the mean of four questionnaire items (e.g., "being a good math student is an important reflection of who I am;" see Appendix for full scale). Student participants evaluated these items for themselves on a six-point Likert scale. This scale was developed as part of the ongoing longitudinal study on motivation in mathematics. The internal consistency was acceptable ($\alpha = .73$) and the items showed excellent fit in a confirmatory factor analysis, with a nonsignificant chi-square statistic, $\chi^2 = .234$ $df = 2$, $p = .89$, comparative fit index (CFI) = .99, Tucker–Lewis Index (TLI) = .99, square-root-mean residual (SRMR) = .006; root-mean-square error of approximation (RMSEA) = .000; 90% CI [.000, .046] (Hooper, Coughlan, & Mullen, 2008; Hu & Bentler, 1999). This scale also established concurrent validity with intrinsic motivation for mathematics ($r = .49$; SRQ-A; Ryan & Connell, 1989) measured in the same sample.

Generalized value of mathematics. Generalized value of mathematics (hereafter 'generalized value') assessed students' abstract value of mathematics. Distinct from attainment value, this scale did not measure perceived importance of mathematics for the individual self, rather the general importance of mathematics broadly (Mickelson, 1990). This concept was assessed through the mean of four questionnaire items (e.g., "I can understand why things that I am learning in math class are important;" see Appendix for full scale). Students rated these items on a six-point Likert scale. This scale was developed as part of the ongoing longitudinal study on motivation in mathematics. The internal consistency was adequate ($\alpha = .78$) and showed good fit in a confirmatory factor analysis, $\chi^2 = 10.31$ $df = 5$, $p = .06$, CFI = .98, TLI = .96, SRMR = .027; RMSEA = .051, 90% CI [.000, .096] (Hooper et al., 2008; Hu & Bentler, 1999). Generalized value was assessed at baseline to more reliably estimate predictors of year-end attainment value.

Cognitive flexibility. Cognitive flexibility was measured through a computerized version of the two-part Trail-Maker Task (TMT-L; Rodewald, Weisbrod, & Aschenbrenner, 2012). This task requires students to remember two sets of rules and consecutively move back and forth between those sets of rules. In the first trial (Part A of TMT-L), students were presented with a screen of 25 numbered circles randomly scattered on the screen. Using a computer mouse, each student had to connect all of the circles in order of least-to-greatest as quickly as possible. In the second trial (Part B; see Figure 1), directly after part A, the students were presented with a screen of 25 circles with half of the circles numbered from one to 13 and the other half of the circles labeled with letters A to L. The circles were randomly scattered. Students had to connect the dots in numerical and alphabetical order, switching between numbers and letters for each move (i.e., 1→A→ 2→ B→ 3→ C) as quickly as possible. For example, in Figure 1, the arrow between letter E and F indicates an error because the correct sequence should be E→6→F. The time score for Part A measured cognitive processing speed and the time score for Part B minus the time score for Part A measured cognitive flexibility. Lower time scores (i.e., quicker processing and execution) indicated higher cognitive flexibility. However, the time scores were reverse coded prior to analyses for ease of interpretability.

Over 25 studies have found the TMT to primarily measure cognitive flexibility but also measure working memory (see Sánchez-Cubillo et al., 2009), demonstrating concurrent validity to the Wisconsin Card Sorting Task ($r = .59$) and the Digit Span Backwards task ($r = .53$). The TMT has also been found to be reliable, as assessed through Cronbach's alpha ($\alpha = .81$; Rodewald et al., 2012) and confirmatory factor analysis (Egle, Debelak, Rodewald, Aschenbrenner, & Weisbrod, 2012). Prior research reported age as a strong predictor of performance (Mitrushina, Boone, Razani, & D'Elia, 2005), highlighting the developmental sensitivity of the measure. In the current sample, this measure had concurrent validity ($r = .23$) with the Tower of London (Kaller, Unterrainer, & Stahl, 2012), a measure of problem solving, which conceptually is an outgrowth of executive functions, including cognitive flexibility and working memory (Diamond, 2013).

Classroom observations. Each mathematics classroom that had student participants was observed for a total of seven cycles between November and March. One cycle consisted of 30 min of observation and coding with two observers in the classroom. The

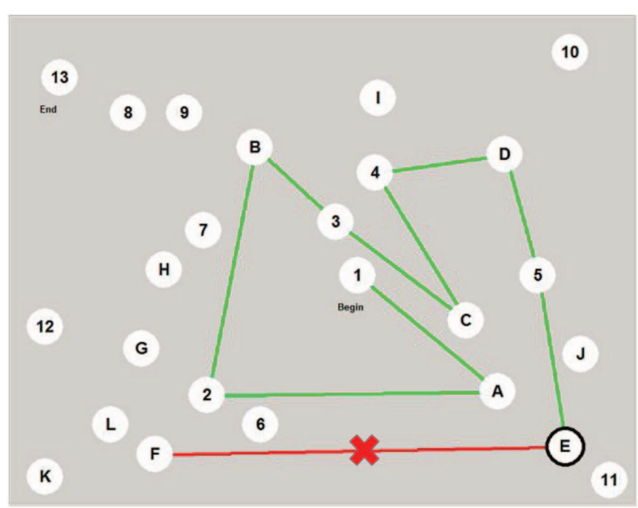

Figure 1. Computerized version of the Trail-Maker-Task (TMT) Part B. This figure is a trial of the computerized TMT Part B, where users must sequentially connect letters to numbers in alternating order. The user had a correct sequence of responses up until the Circle E. There is an error response between Circles E and F. The correct sequence should connect E to 6 and then to F. See the online article for the color version of this figure.

final observation score for each cycle was the average of the two observers' individual scores. Prior to entering classrooms, all observers received extensive training on coding with over 15 hours of instruction and practice, as well as a final assessment to ensure at least 80% reliability.

Teacher math applications. The Mathematics Scan (M-Scan; Berry, Rimm-Kaufman, Ottmar, Walkowiak, & Merritt, 2012) assesses the quality of standards-based mathematics instruction through structured classroom observations. The connections and application subdimension of the M-scan evaluates the extent to which the teacher presents students with opportunities to (a) make connections between math concepts to see math as a web of connected ideas, versus a collection of disconnected procedures, and (b) apply math concepts to their own experiences, the world, and other disciplines. This subdimension was scored on a seven-point scale. There was sufficient interrater reliability ($r = .91$) across nine coders and 31 classrooms. Interrater reliability was computed through a two-way mixed intraclass correlation coefficient analysis with absolute agreement (Hallgren, 2012).

Teacher behavioral and learning support. In an effort to understand the precise effect of TMAs, teacher and classroom qualities related to TMAs were observed and used as statistical controls. Two dimensions from the Classroom Assessment Scoring System-Secondary (CLASS-S; Pianta & Hamre, 2009) were identified as qualities likely possessed by math teachers who were high on TMAs but also reflected qualities conceptually distinct from TMAs; these were teacher (a) productivity and (b) instructional learning formats. Productivity refers to how the teacher manages the classroom so that instructional time is maximized and off-task behavior is minimized. In productive classrooms, teachers are well prepared for the lesson, have effective routines and transitions, and maximize time on topic. Instructional learning formats refer to how the teacher facilitates student learning through clear presentation of the material, providing interesting lessons, and using multiple modalities and strategies during instruction.

The coding protocol described for TMAs applied to these two dimensions as well, including the seven-point Likert scale, coder training, number of cycles per class, and length of cycles. In the current data, teacher productivity and instructional learning formats were highly correlated with one another ($r = .83$), and with TMAs ($r = .70$ and $r = .86$, respectively). To avoid multicollinearity, the mean of the productivity and instructional learning format dimensions were computed to produce one score reflecting *teacher behavioral and learning support*.

Interviews. Follow-up semistructured interviews were conducted with 37 students. Interviewees were selected via maximum variation sampling (Miles & Huberman, 1994) from each of the participating classrooms, with the exception of fifth-grade classrooms.[2] Participants were selected according to engagement (i.e., high, moderate, or low) in their current math class. Selection started during the classroom observation process, described above, with coders noting participation trends across students in their classrooms (e.g., attentiveness, completion of classwork, and answering and asking questions).[3] Once students were selected as representative of the three different strata of engagement, those students' names were brought to their teacher for corroboration. Teachers were then asked to recommend additional students who represented high, moderate, or low engagement. Final interview selection decisions were made to balance the representation of females and males, and Black and Latino students across the five school sites.

The interview protocol was designed to capture students' individual math beliefs, math experiences in the classroom, and the meaning-making of these beliefs and experiences. Students were asked to self-identify their racial-ethnic background and describe their family's culture and family school experiences. Once a student self-identified their racial-ethnic background, the interviewers (and results section in this paper) used that term consistently for that student. Interviewers were trained to use the protocol in a flexible manner (i.e., semistructured) so as to change the sequence and presentation of questions as needed to allow students to tell their stories. Interviewers were also trained to follow up on themes and responses that were particularly relevant to the aims of the study (Kvale, 1996).

Data Analysis Strategy

Quantitative analysis. The first three research questions were assessed through deductive quantitative measurement, in which the hypotheses were tested based on theory and previous research. Bivariate correlations were run to establish relationships among the key variables. Across all of the study variables, missing data ranged from 0.5% to 6.9%. However, an analysis of missing data patterns revealed that the 'missing completely at random' assumption (MCAR) was retained, $\chi^2 = 27.976$, $df = 26$, $p = .360$, and reflected a lack of consistent patterns in how data was missing. Multiple imputation, through the Markov chain Monte Carlo method (Schafer, 1997), was used to replace missing values by generating five data sets and then using the pool of those data sets. The results from the imputed data showed negligible differences from the complete case analysis. Given the nested structure of the data (i.e., classroom and student levels), the appropriateness of multilevel modeling was explored. The intraclass correlation coefficient for year-end attainment value was .042, which indicated that the large majority of variation in attainment value existed between students and very little, 4.2%, between classrooms. Little variation at the classroom level reduces the need for multilevel modeling (Bryk & Raudenbush, 1992). Also, the number of classrooms, 31, and the average number of students per classroom, 13, did not meet the minimum threshold for adequate power in multilevel analyses (Hox, 1998; Richter, 2006). Given these factors, multilevel modeling was not employed.

To address the first three research questions, ordinary least squares multiple regression was used via SPSS 21 (IBM Corp, 2012), with TMAs modeled as the independent variable and year-end attainment value as the dependent variable. Cognitive flexibility was modeled as the moderator on the relation between TMAs and student year-end attainment value. Baseline attainment value, generalized value, student age, and teacher behavioral and learning support were controlled in all analyses. Controlling for

[2] The interview protocol of the broader longitudinal study contained content that was abstract and thus developmentally challenging for younger students to comment on in depth. Initial interview attempts with fifth-grade students showed a consistent lower quality compared with older students. Given time limits and resources to conduct interviews, the PI elected not to continue interviews with fifth-grade students.

[3] I considered these trends as relative based on the classroom, not normative across all classrooms.

baseline attainment value approximated the growth of attainment value over the course of the year as well as predictors of that growth. Controlling for generalized value further improved the precision estimate of attainment value as a personal appreciation of mathematics for the self, versus a general or abstract appreciation of mathematics. Controlling for age accounted for developmental differences in attainment value and cognitive flexibility across grade levels. Controlling for teacher behavioral and learning support allowed for precision on the effect of TMAs, versus other positive teaching qualities.

For modeling the first two quantitative research questions, control variables were entered into the first steps of the regression model, after which cognitive flexibility and TMAs were entered as predictors. In the final step, cognitive flexibility and TMAs were mean-centered before multiplied and entered into the model as an interaction term. Change in R^2 for each step of the model and squared semipartial correlations for each predictor were also reported to ascertain the effect sizes of the models and individual predictors. For semipartial correlations in multiple regression, Cohen (1988) gave standard conventions for small ($f^2 = .02$), moderate ($f^2 = .15$), and large ($f^2 = .35$) effects. The simple slopes of the interaction term were plotted for interpretation and probed for significance against zero. For the third research question, a similar process was conducted; however, year-end cognitive flexibility was the dependent outcome. All other aspects of the model and process remained the same. Confidence intervals (95%) were provided for all the predictors in the final models.

Qualitative analysis. To address research questions four and five, qualitative interviews were used deductively to corroborate the quantitative findings and inductively to allow for the emergence of trends that extended beyond the initial quantitative hypotheses. The interviews were audio-taped and transcribed verbatim. Once transcribed, the interview texts were read multiple times to get an overall understanding of the adolescent's experiences and perspectives. Modified-grounded theory was used to analyze the interviews. Grounded theory allows for the emergence of inductive codes that capture a participant's own interpretation of phenomena as mediated by the social and cultural context (Maxwell, 1996; Strauss & Corbin, 1998). However, modified-grounded theory acknowledges that interpretation of the data is also informed by knowledge of preexisting theory (i.e., the quantitative constructs) that can allow for the emergence of theory grounded in the data (Perry & Jensen, 2001).

To address the fourth research question, the deductive qualitative analysis was guided by the quantitative constructs posed in the first three research questions. Thus, the interviews were coded for students' perceptions of math value, how their teacher applied mathematics to the real world, and how the students attempted to process this information. The coding team examined the patterns in how these codes manifested in the interviews, drew connections between those patterns and developed a value narrative for each interview case. There were eight coders, including the principal investigator. Each interview was coded by a team of three to four coders, first individually and then as a team. The interrater agreement between coders across all 37 interviews was 73.8%. The coders were blind to the students' quantitative scores while reading and developing the value narrative for each student. To determine consistency between the qualitative and quantitative data, the coders then compared the value narrative of each case to that student's attainment value, classroom TMA score, and cognitive flexibility score.

To address the fifth research question, the inductive analysis extracted and defined significant categories and themes that emerged from the data. Open coding was used to transform the data into manageable units by organizing the data from each interview into initial categories, concepts, and properties. These units allowed for the construction of an initial code list that captured key elements within individual interviews and across all interviews. Codes were then grouped to reflect larger thematic and interpretive codes. Each code was defined and examples of each code were provided for reference. Next, a list of codes was generated and combined with those previously identified. These codes were used to develop a code book that summarized the major themes. This code book was used to create a matrix of interpretive codes for each participant (Miles & Huberman, 1994). The matrices were then used to build a cross-case matrix to compare participants on common themes (Strauss & Corbin, 1998).

Results

Quantitative Results

Bivariate correlations between the key quantitative variables were statistically significant (see Table 1). Age was negatively related to attainment value, generalized value, and TMAs, which is consistent with developmental and motivational literature (Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002; Watt, 2004). Age was also positively associated with baseline cognitive flexibility, suggesting older adolescents tended to have higher cognitive flexibil-

Table 1
Bivariate Correlations and Descriptive Statistics of Study Variables

Variable	*M* (*SD*)	1	2	3	4	5	6	7	8
1. Age in months	155.88 (19.12)	—	−.09	−.28**	−.25**	−.16**	−.27**	.15**	.10
2. Teacher behavioral & learning support	4.47 (.82)		—	.09	.01	.09	.81**	.16**	.11*
3. Generalized value	4.73 (1.08)			—	.50**	.43**	.12*	−.03	.00
4. Baseline attainment value	4.46 (1.17)				—	.56**	.08	−.03	−.01
5. Year-end attainment value	4.26 (1.29)					—	.16**	.00	.14**
6. TMAs	2.57 (.64)						—	.12*	.06
7. Baseline cognitive flexibility	44.60 (12.06)							—	.47**
8. Year-end cognitive flexibility	52.32 (8.93)								—

Note. TMAs = Teacher math applications.
* $p < .05$. ** $p < .01$.

ity scores compared with younger adolescents. Sample means revealed attainment value declined over the school year from baseline to year-end, while cognitive flexibility increased. TMAs were also positively correlated with year-end attainment value. Overall, TMAs were low across classrooms ($M = 2.5$) and tended to be lower among older students, indicating upper grade-level classrooms were less likely to engage TMAs. Despite diversity in academic reputation across the five schools, school differences in TMAs only ranged from 2.31 to 2.76 on a seven-point scale, with the two magnet schools positioned around the center of this range distribution.

The ordinary least squares regressions contained six models for entering the control and main variables. Age was entered in the first model as a significant negative predictor of year-end attainment value (see Table 2). In the second model, baseline attainment value and generalized value positively predicted year-end attainment value, controlling for age. Third, teacher behavioral and learning support was a significant positive predictor, controlling for the previous variables. In the fourth model, cognitive flexibility did not predict year-end attainment value. TMAs were a positive predictor in the fifth model, indicating that TMAs significantly predicted growth in attainment value over the school year, controlling for teacher behavioral and learning support, baseline attainment value, generalized value, and age. The results of this fifth model addressed the first research question. In the sixth model, the interaction between TMAs and cognitive flexibility predicted year-end attainment values controlling for all the variables entered in the previous models. This final model explained 36% of the variation in year-end attainment value; however, squared semipartial correlations revealed TMAs and the interaction effect only explained a combined 2% of the variation in year-end attainment value.

The interaction effect was plotted and probed (see Figure 2), illustrating that students with high cognitive flexibility scores (i.e., the dashed line) who were also in classrooms with frequent TMAs experienced more growth in year-end attainment value. Conversely, the slope for students who scored low on cognitive flexibility (i.e., the solid line) was relatively flat and not significantly different from zero. Thus, there was no significant difference on attainment value growth for these students regardless of whether they were in classrooms with high or low TMAs. The results of this interaction effect addressed the second research question.

To examine the third research question, year-end cognitive flexibility was modeled as the dependent outcome. The control variables and predictors were entered in the same stepwise fashion as previously described when year-end attainment value was the dependent outcome. In the sixth and final model, only baseline cognitive flexibility positively predicted year-end cognitive flexibility (see Table 3). Further, the interaction between TMAs and baseline cognitive flexibility did not predict year-end cognitive flexibility.

Qualitative Results

Deductive findings. Beginning deductively, I used the second quantitative hypothesis to organize the 37 interview transcripts for qualitative corroboration. The second quantitative hypothesis posed that students with stronger cognitive flexibility who were also in math classrooms high in TMAs were more likely to

Table 2

Standardized Coefficient Predictors on Year-End Attainment Value With Interaction Effect

	Year-end attainment value (Y)													
	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6			Squared semipartial correlation
Predictor	B (*SE*)	β	B (*SE*)	β	B (*SE*)	β	B (*SE*)	β	B (*SE*)	β	B (*SE*)	95% CI	β	
Age in Months	−.01 (.00)	−.16**	.00 (.00)	−.01	.00 (.00)	.01	.00 (.00)	.01	.00 (.00)	.02	.00 (.00)	[−.01, .01]	.00	.00
Baseline attainment value	—		.48 (.05)	.44**	.49 (.05)	.45**	.49 (.05)	.45**	.48 (.05)	.44**	.48 (.05)	[.37, .59]	.44**	.14
Generalized value	—		.26 (.06)	.21**	.25 (.06)	.21**	.25 (.06)	.21**	.25 (.06)	.21**	.25 (.06)	[.13, .37]	.20**	.03
Teacher behavioral & learning support	—		—		.14 (.06)	.09*	.13 (.07)	.08	−.07 (.11)	−.05	−.07 (.12)	[−.30, .16]	−.05	.00
Baseline cognitive flexibility	—		—		—		.00 (.01)	.02	.00 (.01)	.02	.01 (.01)	[−.01, .02]	.05	.00
TMAs	—		—		—		—		.32 (.16)	.16*	.34 (.16)	[.03, .65]	.17*	.01
TMAs × Cognitive Flexibility	—		—		—		—		—		.02 (.01)	[.00, .03]	.11*	.01
	$R^2 = .027$		$R^2 = .335$		$R^2 = .342$		$R^2 = .342$		$R^2 = .350$		$R^2 = .360$			
F for change in R^2	10.32**		83.47**		4.19*		.17		4.31*		5.63*			

Note. TMAs = Teacher math applications; CI = confidence interval.
$^{*} p < .05$. $^{**} p < .01$.

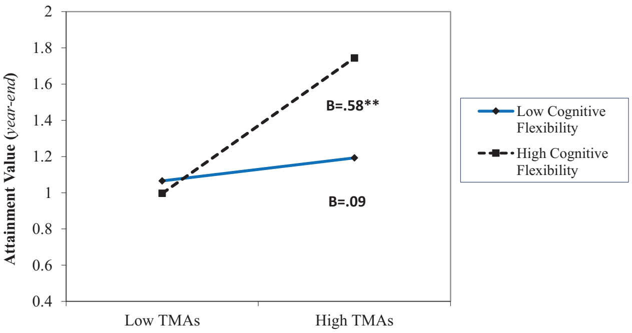

Figure 2. Interaction plot of teacher math applications by cognitive flexibility with simple slopes. Dashed line represents students who scored one standard deviation above the mean on cognitive flexibility. The solid line represents students who scored one standard deviation below the mean on cognitive flexibility TMAs = teacher math applications. ** $p < .01$. See the online article for the color version of this figure.

experience growth in attainment value. Thus, the interview texts were coded for all forms of academic value substantiated by previous research. These included intrinsic value (18% of value codes), attainment-internalized value (10% of value codes), utility value (23% of value codes), abstract value (25% of value codes), uncertainty (8% of value codes), and negativity (16% of value codes). The value narrative from each student was analyzed and compared with the second quantitative hypothesis for consistency.

Fourteen (38%) of the student narratives were *highly consistent* with the second quantitative hypothesis, 10 (27%) were *partially consistent*, six (16%) were *inconsistent*, and seven (19%) *could not be determined*. To be considered 'highly consistent,' the interviewee's qualitative narrative needed to illustrate relationships between all three quantitative constructs, (a) attainment value, (b) TMAs, and (c) cognitive flexibility in a way the second quantitative hypothesis would predict. In other words, relationships between all three concepts within the value narrative had to be evident and had to align with the second quantitative hypothesis (below see interview excerpt from Shae as an example). 'Partially consistent' meant that one of the three quantitative variables did not corroborate the student's qualitative narrative in a way the second quantitative hypothesis would predict. For example, if a student's narrative described low TMAs in their classroom but the objective observations indicated high TMAs, this case would be categorized as partially consistent as long as all of the other indicators aligned. "Inconsistent" meant that more than one of the three quantitative variables did not corroborate the student's narrative in a way that the second hypothesis would predict. For example, a student might have described their growing value of mathematics because his teacher is always connecting math to the real world. However, if this student's year-end attainment value was lower than his baseline and his classroom was observed as moderate or low on TMAs, this case would be categorized as inconsistent. 'Cannot determine' indicated a lack of information in the narrative or quantitative variables. This often resulted from insufficient detail in the interview, inability to interpret substantive meaning from the interview, or missing data for that individual student.

Below is an illustration of a value narrative highly consistent with the second quantitative hypothesis. Shae was a ninth grader

Table 3
Standardized Coefficient Predictors on Year-End Cognitive Flexibility With Interaction Effect

Predictor	Year-end cognitive flexibility (Y)													Squared semipartial correlation
	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6			
	B (*SE*)	β	B (*SE*)	β	B (*SE*)	β	B (*SE*)	β	B (*SE*)	β	B (*SE*)	95% CI	β	
Age in months	.05 (.02)	.11*	.05 (.02)	.12*	.06 (.03)	.13*	.02 (.02)	.04	.02 (.02)	.05	.03 (.02)	[−.02, .08]	.06	.00
Baseline attainment value	—		.17 (.46)	.00	.09 (.45)	.01	.12 (.41)	.02	.10 (.41)	.01	.11 (.41)	[−.69, .91]	.02	.00
Generalized value	—		.18 (.50)	.02	.10 (.50)	.02	.13 (.44)	.02	.14 (.44)	.02	.15 (.44)	[−.72, 1.03]	.02	.00
Teacher behavioral & learning support	—		—		1.48 (.54)	.14**	.60 (.50)	.05	−.04 (.87)	−.01	−.04 (.87)	[−1.76, 1.66]	−.01	.00
Baseline cognitive flexibility	—		—		—		.33 (.04)	.46**	.33 (.04)	.46**	.33 (.04)	[.25, .40]	.45**	.17
TMAs	—		—		—		—		1.05 (1.16)	.08	1.02 (1.16)	[−1.25, 3.30]	.08	.00
TMAs× Cognitive Flexibility	—		—		—		—		—		−.04 (.05)	[−.14, .06]	−.04	.00
	R^2 = .012		R^2 = .013		R^2 = .033		R^2 = .228		R^2 = .230			R^2 = .231		
F for change in R^2	4.50**		.097		7.40**		90.24**		.827			.768		

Note. TMAs = Teacher math applications; CI = confidence interval.
* $p < .05$. ** $p < .01$.

who identified as a Black American female. The high school she attended was predominantly Black and has consistently been ranked among the lowest in the state for consecutive years. Despite this, Shae was in an algebra one classroom where her math teacher gave frequent and high-quality TMAs during in-class instruction. Shae also had a high cognitive flexibility score, in the 94th percentile of this study's sample, which may reflect her capability to 'think outside the box' or across multiple dimensions fluidly. Based on the second quantitative hypothesis, Shae would be a student predicted to experience growth in attainment value over the year. Below is an excerpt of her interview that reflected this growth and demonstrated her as 'highly consistent' with the second quantitative hypothesis.

> First I didn't like it [math] at all. I really didn't, but now I feel as though it's not as hard. I do not know maybe it's the teacher, we didn't have a bond like I have with [Teacher Name]. But I feel like I like math now. . . . I do not know she just . . . she shows us a lot, but I do not know how to explain it, because when you see it, it's like, 'whoa' but then when you gotta put it in words yourself it's like I do not know at the time period. But yeah, I think she's just trying to prepare us for when we get older. We can understand more and we can know how things go in the real world because I think we're gonna need to know a lot about numbers in the real world.
>
> Oh, I'll make a graph out of this because I'm riding my skateboard. Oh, I'mma ride my bike down the hill so I'mma make this turn into this in math ways. I would think I'm just joy riding. I wouldn't think of math, as the first thing that pops in my head. So it's very exciting to know you could do this. And then when she explained it I was like oohhh, like I didn't even think of that, like when something hit me I was like oh I could of thought of this . . . but maybe it's teaching you how to think outside the box. (Shae, ninth grade, self-identified African American female)

In this text, the three quantitative variables aligned and corroborated her narrative, reflecting (a) her growth in attainment value, (b) the presence of TMAs, and (c) her cognitive processing (i.e., flexibility). Shae has an internalized value of math, which for her reflects growth in attainment value. She expressed math was never a topic she cared for much, but now she was beginning to change due to her bond with the teacher and the teacher's use of TMAs. She seemed to recognize these messages were powerful for her seeing math in a new way. Although she could not articulate the real-world application in her own words, the TMAs had meaning for her thinking about her future and her present. For example, she understood the connection between riding her bike down a hill and graphing rate of change equations. Shae even mentioned how TMAs provided practice in "teaching you how to think outside the box," which might suggest TMAs can support cognitive flexibility.[4]

Shae's excerpt was an example of a 'positive' case of consistency with the second quantitative hypothesis, reflecting high TMAs and growth in attainment value. However, there were also negative cases[5] that were also highly consistent with the quantitative hypothesis, reflecting negativity or uncertainty about the value of mathematics and rare instances of teachers discussing mathematics for the real world. Among highly consistent and partially consistent cases, negative examples represented 43% and 80% of those cases respectively.

Inductive findings. In addition to corroboration, the value narratives also possessed emergent themes that extended beyond the initial quantitative hypotheses. Three overarching themes were identified during inductive analysis: (1) *utility orientations* (with subthemes (1a) *abstract future benefits* and (1b) *math is for practicing smartness*), (2) *alternative messengers,* and (3) *resisting stigma and protecting collective identity.* Unlike quantitative reporting, qualitative presentations of findings are often reported in ways that show the relationship between the raw data and the interpretation of that data (Anderson, 2010). Thus, illustrative quotes that best represent a theme are presented along with an interpretation of that text to underscore important elements within the data simultaneous with interpreting its meaningfulness and connection to the research literature. The present findings are also reported in this manner.

Theme 1: Utility orientation. Nearly all 37 interviewees discussed the utility of mathematics for present and future goals. Although these narratives were consistent across the five schools, they varied on a few internal dimensions, such as positive versus negative utility, academic versus personal utility, and the quality of detail in the utility examples. Considering such patterns, two consistent subthemes were identified, entitled: (1a) abstract future benefits and (1b) math is for practicing smartness.

Subtheme 1a: Abstract future benefits. This subtheme was one of the most cited and clear themes that emerged from the data. Twenty-one (out of 37) students discussed the potential for mathematics to have utility value for future classes, college, or certain jobs (e.g., math teacher, engineer) yet struggled to find its utility in their present lives. Several students directly admitted that mathematics was not currently important for their lives. This narrative was most prevalent among students who expressed a lot of negativity about mathematics; however, even students with positive attitudes toward mathematics struggled to articulate its relevance for their current lives. Further, throughout this subtheme, students tended to indicate more performance goals (Ames & Archer, 1988; Sullivan, Tobias, & McDonough, 2006) versus mastery goals, meaning they were more likely to articulate a value of math performance versus a value of math learning. Overall, instances of abstract value, utility value, uncertainty, negativity, and performance goals were intertwined across the statements within this theme. As an example, consider the following statement:

> I do not think that math is useful outside of the classroom, but it's not like- - of course it is! Like every day with money. Count money, going to the store and buying this and buying that. But I do not think certain things that you get taught every day is for the outside world unless you tryin' to get a certain job. But as a regular freshman coming out of school? I do not understand what tables and graphs have to do with going out of school and living everyday life. I understand the future when you get certain jobs and stuff, but for now I do not think it's worth it. Well, yeah basic numbers. But, the graphs, the tables, all of that I do not think that's useful for everyday life as in now. (DaShane, ninth grade, self-identified African American male)

Here, DaShane attempted to balance his candid beliefs about the

[4] For another example of interview text that suggested TMAs can support cognitive flexibility, see the text by Jackson in Theme 1b.

[5] See the interview text from DaShane in Theme 1a as one example of a negative case that was highly consistent with the quantitative hypothesis.

irrelevance of mathematics along with the practical benefits of mathematics for counting and money purposes. Beyond basic counting, he conveyed a strong notion of abstract value, where mathematics was important for certain jobs, likely meaning prestigious jobs, but was not very important for his everyday life as a high school freshman in his neighborhood. The school DaShane attended was located in a predominantly Black part of the city with the highest rates of crime and poverty. Thus, his testimony given within the context of this environment aligns with Mickelson's (1990) conclusion that socially marginalized adolescents may think of their education as important for a life beyond their immediate reality, possibly a life they do not believe they will have the opportunity to attain. However, it is important to note that students across all five schools, even the magnet schools, conveyed similar sentiments. Many adolescents gave examples that were abstractions about a future life or jobs in which they do not see themselves. For example, Nina attended one of the state-ranked magnet schools, but said:

> I feel like it is important and like some [things] that the teachers teach is not that important. Like I do not see why we need equations in life, graphing. I mean I understand if you want to work with like math: a math teacher, engineering, or like a scientist you need to know that stuff. But I do not want to deal with nothing that has to do with math. (Nina, ninth grade, self-identified Puerto Rican & Dominican female)

Altogether, it seemed that some students acknowledged mathematics was important for very basic things (e.g., money and counting) as well as very prestigious things (e.g., becoming an engineer), but struggled to find meaning for their everyday lives. Some researchers argue this view may not be a result of deficit thinking by students, rather it is cultivated by systems in education that perpetuate inequalities in the perceived possibilities of mathematics for Black and Latino students (Apple, 2013; González, Andrade, Civil, & Moll, 2001; Schoenfeld, 2002). In addition to Nina's quote above, she mentioned that there was little that teachers did to convey the importance of mathematics, even in her magnet school which prides itself on its strong math scores compared with other district schools. She said, "They don't really talk about it. We don't really talk about like life, I don't know. Like if we're going to have a conversation about that, it's not often. Maybe like once in a blue." She even mentioned going so far as to do her own research on the Internet and ask her mother about the importance of mathematics due to the lack of opportunities to have her curiosities addressed in the classroom.

Ultimately, this might suggest that the glass ceiling that Black and Latino adolescents perceive in mathematics and education more broadly (Mickelson, 1990) is likely more than just a figment of their imagination. The lack of opportunities for these adolescents to have mathematics presented in a way that challenges their conceptions of what they believe is possible in their present world (Berry, 2003; Schoenfeld, 2002) inherently restricts them from realizing the boundlessness potential of mathematics for their future world. Such neglect in engaging the meaning of mathematics may prime Black and Latino students for a social experience of marginalization in postsecondary education and the workforce.

Subtheme 1b: Math is for practicing smartness. Alternatively, there were a smaller subset of students, approximately eight, who made profound meaning of their mathematical experiences for their own personal and social development. Students in this theme were represented across all five schools. One student, Paul, said, ". . .it's like math like helps me with that stuff because without math I wouldn't be this smart!" By "that stuff" Paul was referencing how he used mathematics to make decisions about how much he should tip the waiter at his favorite restaurant, TGI Friday's, which was recently built in his community. He seemed to take pride in knowing that he never overtips or undertips because his math skills helped him be socially smart. Like Paul, adolescents who expressed this subtheme held the idea that mathematics gave them practice at behaving like a smart person, and smart people always get ahead in life because they are better able to navigate their world. Seldom did these adolescents talk about the importance of mathematics in an academic or performance sense (e.g., grades, a job, future classes), rather they primarily talked about it for personal prowess, tackling life problems, and making the world a better place. Two adolescents used bodybuilding analogies to suggest how mathematics presented challenges that allowed for growth once these difficulties were overcome. Intertwined within these narrative statements were notions of attainment, utility, and even abstract values simultaneously; however, throughout these narratives persisted the idea that knowing mathematics allowed opportunities for personal development and agency.

Consider this exchange between Jackson, a ninth-grade self-identified Haitian American male, and the interviewer. Jackson discusses how math pushed him to think deeply and become savvy in using information to solve classroom problems and world problems.

> Jackson: It's that thing [math] that always gets you thinking cause, I do not like thinking much but math . . . when you get a problem it makes you think critically and pushes you.
>
> Interviewer: What does thinking critically mean to you?
>
> Jackson: So you have what you've been taught so you're gonna use that. So you're given a problem that you've never seen before or that's similar but you do not know it so you're gonna use what you were taught and infer . . . Once you do something like a math problem or you [are] just really into math you'll probably be able to have better thinking skills or something and you're able to think quicker and faster and sometimes you're able to find a new way to solve something . . . it's just being able to put what we do into real world problems . . . yeah like what we can do to better the world that we live in. That's what I'm really interested in.

Jackson went on to discuss the importance of mathematics for creating new technology, safer cars, and better weapons for national defense. However, it is also interesting to note that when asked about the importance of mathematics currently, he admitted that it was not very important now but for the future. He seemed to convey the utility of mathematics as something that was incubating; to be realized in the future, although for now it might not be as important. This highlights a similarity with the abstract future benefit subtheme, in that many adolescents struggled with the present value of mathematics. However, the adolescents who espoused the practicing smartness subtheme seemed to understand mathematics as building prowess that would make them powerful agents for navigating their world. Finally, narratives within this subtheme sometimes showed more traditional aspects of attainment value (e.g., I see myself as a math person; Wigfield, 1994); however, more often these adolescents revealed an agency in knowing math, which aligned with more culturally relevant conceptions of mathematics identity (e.g., to change the conditions of one's life; Martin, 2012, p. 57). This idea is revisited and elaborated on in the theme on resisting stigma and protecting collective identity.

Theme 2: Alternative messengers. This theme illustrated how messengers other than the mathematics teacher can play an important role for adolescents understanding the real-world importance of mathematics. More than one third of the adolescent interviewees mentioned family relatives (e.g., uncles, grandparents) or other teachers (e.g., science teacher) who provided consistent messages regarding the importance of mathematics. In many cases these alternative messengers were majoring in postsecondary mathematics, enjoyed math, or used numbers often in their work. One student mentioned how she would watch home repair TV shows with her uncle as he would talk about the mathematical nature of the projects. Another student talked about his grandfather, not only as a math support and model, but also for providing a real-life context that helped him understand the concept of adding negative numbers. He explained,

> He was born in Africa. And so he was really good at school. . . . He went to college in England, and he went to another college in Russia. And so now . . . he became a professor in George Washington University for Calculus. . . . And then now, he works at the UN.
>
> Yeah, he's the best at math in my family. He's the go-to guy other than my brother . . . like I remember the best thing he gave me, I was having trouble with negative numbers, cause like addition and like there's certain rules, it changes. And so he was like, "Okay, so envision the steps at our house," cause there's steps from the second floor to my grandparents' house, but they moved out of that house some time ago. . . . So he's like, "Envision the stairs to our bedroom. So say you do $5 + -10$, right?" So he was like, "You have to go up 5 stairs, but then you have to go back down 10 stairs, so you end up at -5." And so, yeah, so he helped me around that. (Jaden, seventh grade, self-identified multiracial male)

Here the grandfather could have provided a number line, which many teachers often do to model the concept of adding negative numbers. However, he took this one step further and used a familiar context (i.e., the stairs in their home) to represent the same idea as a number line. Because he used this number line in a personalized context (Walkington, 2013), the adolescent was more likely to understand and remember the application.

In nearly every case, the alternative messenger knew how to apply the mathematical concept toward the adolescent's personal aspirations or interests (e.g., an engineer, a business woman, getting into college). This is consistent with previous research that underscores how family investment in mathematics can be meaningful for helping adolescents understand the power of mathematics for their lives, their culture, and their community (Martin, 2006). However, critical scholars in math education go a step further, describing how family support of the Black child's struggle for developing mathematical literacy represents more than just the academic or social interests of the child; rather, it exemplifies a greater collective struggle against racial subjugation and discrimination (Leonard, Brooks, Barnes-Johnson, & Berry, 2010; Martin, 2006). Leonard and her colleagues (2010) suggested that mathematics is not a 'race-neutral' subject; therefore, Black and Latino students can receive subtle messages in school and society that signal (McKown, 2013) a marginal view of the limited future accessible to them due to their racial group or the neighborhood of their upbringing. In light of this, racial and math socialization messages from the alternative messengers of Black and Latino children become imperative to buffer against stigma. Shayla, a student who attends one predominantly Black high school that has consistently been ranked among the worst in the state, discussed messages from her parents that helped her counter stigma about her potential and her future.

> Because growing up my parents always pushed me about education so I started to get to think that I do not need them to push me, I'll do it on my own. So, A's and B's were like the best thing ever, the first time I got a C, I nearly cried. Like I was about yay-close from losing my mind because I do not like C's cause C's are average. I do not want to be average. I want to be top of the line. . . . I want to be that kid that be like oh you came from [neighborhood name] but you going to this college . . .
>
> They would tell me, when you look in the mirror you're Black. You cannot change that and there is already enough people that do not think of my culture as a good thing . . . so what I realized is that you have to work 10 times harder, knowing your circumstances. Like it's not a bad thing about being Black, it's not, that's one of the best things to be in life. Like it's good but you realize that you have to work 10 times harder because people is judging you off of what they think like a stereotype or a statistic. (Shayla, ninth grade, self-identified Black female)

In this excerpt, there was a clear internalization of parental messages that developed into motivated resistance. Although mathematics was not mentioned explicitly in this excerpt, within the next theme, it becomes clear how these messages translated into a growth mindset (Yeager & Dweck, 2012) for Shayla, even when she encountered difficulties in her math performance.

Theme 3: Resisting stigma and protecting collective identity. This final theme underscored a racialized perspective on the previously described themes. The theme was represented by four Black adolescents, including Shayla, who might be considered unique in their conceptions of the world, their place in it, and their understanding of mathematics. These students brought a critical consciousness to their perceptions of stigma. They resisted pejorative perceptions about their race and attempted to rewrite these scripts through their persistence in mathematics, their sociopolitical understanding of racial stereotypes, and supportive messages from family members. Altogether, these students embodied a healthy intersection of racial and mathematics identities that translated into a resilient mindset. Three dominant patterns emerged within this theme: (a) resisting stigma as a source of motivation for math persistence (Andrews, 2009), (b) collective identity and struggle (O'Connor, 1997), and (c) possessing a growth mindset for coping with math frustration. Patterns (a) and (b) are reflected in the following statement:

> I just want to get good grades so it could make the school look better and I'mma look better. So like if you just look at the school before it probably could've been bad and students here do not really learn. But if we keep working, we keep getting our grades up it's going to make the school better statistically. So it does mean something to me as African American children. . . . Because if the school looks better then the students would too. Because the first thing they'll look at is the students that attend the school and they look at them and be like ooh well they have their thoughts but at the same time we have good grades and so they cannot doubt us by how we look if we getting our work done. . .

> I try to go up to the board. When I go up to the board the majority of the class looks because they know that I'm going to say something. So I go up there and I use it as an opportunity to reach those who do not understand or understand a little bit . . . Cause it may be some people that wants to know but like probably talking before or not here at school, but they want to know. So, when I go up there they can hear it from a student's perspective and for the kids who kinda sorta knows what's going on I could just like clarify. Even if I get it wrong, I still try to, I try to teach the students. (Kasir, ninth grade, self-identified African American male)

It is important to note the four students in this subtheme attended the same high school which, as mentioned previously, had consistently been ranked among the lowest in the state, was homogenously Black (~90%), and had a poor academic and behavioral reputation within the district. This intersection of a predominantly Black and historically failing school created a stigmatized association between race and achievement, reflecting poorly on Black students in the district. Thus, the four students in this subtheme strived to be exceptional to resist stigma placed on the school and thus the Black community in the district. Kasir not only conveyed this resistance mentality, he also grounded his stance within a collective orientation. His achievement was not only for self-preservation, but also for garnering respect for his school and the Black community. In his algebra class, he liked the challenge of visiting the board to solve problems in front of the class, but was also concerned with engaging his peers and pulling them up with him.

Unlike Kasir, not all the students in this theme had high efficacy in mathematics. Despite this, their resistance mentality translated into different forms of persistence and coping strategies, even when they were frustrated in mathematics. This coping and growth mindset (pattern C of this theme) was demonstrated by Shayla, who was introduced in the previous theme. She noted,

> For math, I would look at a problem and then I would make a problem similar to it and then turn the notes face down and then try to do it and see if I get it right . . . Like I would read things over and over again so I can comprehend it . . . I feel like it's annoying but then I feel like that is something that I have to do to get through cause everybody isn't always the best at everything they first try to do in life. So I have to push myself, I have to learn how to be able to pick up things and work with things and know how to understand because if I do not then I am just going to be- that's always going to be my excuse, oh math was never my subject. Like I do not want to have an excuse, I want to be able to be like math was not my subject at some point of time but I pushed myself and I was able to overcome that. I was able to do better. (Shayla, ninth grade, self-identified Black female)

This is only an excerpt of the many cognitive and behavioral strategies Shayla employed in her honors algebra one class. Interestingly, she mentioned that she only gave this extra effort in mathematics because she felt like it was her weakest subject. Thus, Kasir and Shayla represented important differences in terms of math track (i.e., honors vs. nonhonors) as well as math efficacy. However, despite these differences, the three patterns of (a) resisting stigma as a source of motivation for math persistence, (b) collective identity and struggle, and (c) possessing a growth mindset for coping with math frustration were consistent markers for these adolescents' perspectives on mathematics, themselves, and their community. Due to the overrepresentation of Black students at low-performing and stigmatized schools in the current data, the adolescents who substantiated this theme were all Black. Latino adolescents also discussed central elements of this theme (patterns a through c), although not to the strength and consistency of these four Black adolescents. However, these results likely apply to stigmatized Latino adolescents as well.

Given the buoyant strength of these students in the face of social and personal challenges, a natural question that arose was how did these students develop such resilience and perspective? Although the current data are incomplete to answer this question, it may provide some clues. As was discussed in the alternative messenger theme, racial and mathematical socialization seemed to be central in the development of these adolescents and their perspectives. For example, we get a sense of Kasir's intellectual, familial, and racial pride in this statement,

> Yeah my family, they like always been real good in math . . . my great grandmother she was a math teacher so she taught my father and that's why my father's so smart because he had her to help. It was like we always had—We was always good at math in our family.

In addition, the notion of collective identity and struggle may develop as these adolescents begin to truly see their existence and their actions as a contribution to the Black experience in America. Development of such collective identity may be a developmental issue, as the younger adolescents tended to talk about their race as a descriptor, category, or marked by static stereotypes. Conversely, these four older adolescents saw their race as making them part of a legacy that they had the opportunity to contribute to; and perhaps a legacy that has historically embodied a resilient spirit. Shayla said,

> We are trying to make something out of nothing. We live in a community where there is really nothing so when we sit here trying to do stuff and you know try to be a positive effect on it, they always look toward the bad. . . . I feel like my community is, is crazy! People is dying. . . . Like it's always a crime being committed . . . but then when I realize I am just as part of that community as anybody else. So I feel like what I should be trying to do is making the change for myself and my community. Like trying to change things. . . .

Altogether, this theme revealed an alternative form of math value; one enacted through resistance, where the individual used mathematics to highlight their exceptionality as a challenge to prevailing notions of deficiency regarding their race and culture. Through mathematics and socialization messages from others, these students developed personal pride, seeing themselves and their actions as contributing to a legacy of their people that is still evolving.

Discussion

The current study applied concurrent mixed methods to examine how urban Black and Latino adolescents understand the relevance of mathematics for their lives. Most notable across both the quantitative and qualitative data was that TMAs matter to these adolescents, and they matter in a variety of ways. Mathematics teachers who consistently showed the interconnections of mathematical concepts and provided concrete examples of how those concepts applied to the real world (i.e., TMAs) had students who were more likely to grow in valuing mathematics over the course of the year. Although the effect size for TMAs was small numerically (f^2 = .01; Cohen, 1988), this finding is still meaningful practically. Consider that the current data shows attainment value declines by 4.5% (0.20 scale points) from baseline to year-end, a trend generally substantiated for adolescents by previous research (Jacobs et al., 2002; McCoy, 2005). Despite this decline, the current data also show that a one-point increase in TMAs (M = 2.5 on seven-point scale) correlates to a 7.5% (0.32 scale points) increase in students' year-end attainment value, negating the initial decline and marking a small increase over the school year. Thus, even a modest upgrade in TMAs can potentially make a noteworthy impact on well-known and consistent declines in adolescents' value of math.

The qualitative interviews also corroborated the quantitative trends. The personal narratives of the adolescents evidenced that they listened to and internalized the TMAs of not only their math teachers, but the applications of alternative messengers alike. Many adolescents directly connected these messages to their own attainment value of mathematics; and adolescents who did not have teacher or alternative messengers were more likely to voice frustration, uncertainty, or negativity in valuing mathematics. Further, cognitive flexibility moderated TMAs' association with growth in attainment value, showing that adolescents with higher cognitive flexibility scores essentially took more meaning away from TMAs toward negotiating their own math identity (i.e., attainment value). Altogether, these findings revealed that teacher efforts to apply mathematics to the real world were meaningful for adolescents; but also, general maturation in cognitive flexibility helped adolescents process and internalize these applications for their own sense of value for mathematics.

Although TMAs predicted growth in attainment value, it did not predict growth in cognitive flexibility after controlling for other variables. This result contrasted with the qualitative data, whereby some students clearly mentioned that having their teachers apply mathematics to the real world pushed their thinking, or helped them think outside the box. Greater consistency between the quantitative and qualitative data here would have suggested that teachers, through their TMAs, can support their students' cognitive flexibility development. Some researchers have already shown that executive functions do not exist purely as an internal maturational phenomenon; rather they can be externally supported by adults (Diamond & Ling, 2016). However, this work is mainly in early childhood populations. Future research will need to investigate whether this extends to adolescence and if so, which precise instructional qualities predict growth in adolescent cognitive flexibility.

In addition to generally corroborating the quantitative trends, the qualitative interviews also unearthed patterns that extended beyond the initial hypotheses. The interview data revealed the various forms of adolescent meaning-making in mathematics, both traditional and racialized. One traditional form could be described as utility orientation, where many students discussed the importance of math for some future goal (e.g., college, jobs, GPA). This is quite a popular form of achievement values, consistent with previous research (Wigfield & Eccles, 1992; Wigfield & Cambria, 2010). However, the current data also underscores perceptions of the utility of mathematics as contextualized. These adolescents perceived the usefulness of mathematics as embedded within the immediate context of what they believed to be possible for themselves. Mathematics may be important broadly but is not of personal importance for those who believe it is only useful for social opportunities in life that they'll never have the chance to experience (e.g., "I don't think certain things that you get taught every day is for like the outside world unless you trying to get a certain job" - DaShane). Therefore, a traditional notion of utility value, which has shown to be predictive of important outcomes in mainly white and middle-class populations, may manifest differently for urban Black and Latino adolescents who may recognize the confines and challenges of their social and economic environment.

Another traditional form of meaning-making in the present data is the everyday revelations that adolescents begin to see in their current lives, such as when Shae recognized riding her bike down the hill was a rate of change equation that she could graph. This may characterize an internalized notion of value (Ryan & Connell, 1989), which has been studied for over three decades. However, despite this long-existing knowledge, the current sample of adolescents communicated racialized insights that are less well known in the traditional value literature. Some students revealed an interesting intersection of racial and mathematics identities that provided a sense of agency, enacted through a collective struggle and resistance mentality (O'Connor, 1997). Kasir communicated this mindset aptly in his statement, ". . .they can't doubt us by how we look if we getting our work done." In essence, we learn that one's value of mathematics can transcend future aspirations of college and career or everyday revelations of mathematics in the real world. Moreover, valuing mathematics can exist as a psychological stance for affirming desired ideals (e.g., Black intellectual pride & family legacy in Kasir's case) while also challenging stigma (e.g., social and academic perceptions about Shayla's school and Black people more broadly). This may reflect a 'race-reimaged' (DeCuir-Gunby & Schutz, 2014) conceptualization of attainment value that conveys the idea that marginalized adolescents can self-actualize and become empowered through mathematics, beyond simply seeing themselves as a 'math person' or wanting to engage in a math-related career.

Limitations

While interpreting these results, it is also important to understand the limitations of this work. First, this study is a nonexperimental design, thus TMAs could not be completely isolated from the influence of confounding or extraneous variables. Although I attempted to address this limitation through several powerful teacher and student controls, other potential covariates (e.g., cognitively demanding math tasks) might exist that could affect how the study variables related to one another. Therefore, in describing the results of this study, I was sensitive to do so in terms of relationships, associations, or predictions, and not to convey casual inference. Despite these limitations, the statistical controls and temporal elements of the quantitative modeling remain noteworthy for illustrating the relation between key variables that predict growth in attainment value among urban Black and Latino adolescents. Future research might consider a counterbalanced experimental design to isolate the exclusive effects of TMAs on adolescent outcomes.

Second, the sampling for this study was not random, but relied on students' and their parents' volunteerism at the five school sites. Thus, chronically absent, disengaged, or overly disciplined (e.g., expelled) students may be underrepresented in this sample. Overall, this limits generalizability to the greater population within the district. However, it is encouraging to note that the racial demographics of the sample were generally consistent with the school district population. Further, the five schools represent important differences in racial composition, school culture, and academic rigor. This diversity across schools helps account for the differences in school experiences students may encounter throughout the greater district.

Third, the observational measure for TMAs (M-Scan; Berry et al., 2012) has been found to be valid and reliable (Ottmar et al., 2015); however, it may not necessarily account for the cultural authenticity of the applications. In other words, although the observational coding takes into account frequency and depth of the applications, it may not account for whether the applications used are the most appropriate (i.e., tailored) for the students in each specific classroom. Despite this methodological limitation, trends in similar research (Turner et al., 2009; Walkington, 2013) corroborate the overall direction of this study's findings. Further, a standardized observational measure of the cultural authenticity of TMAs does not currently exist to my knowledge.

Finally, some might consider the TMT-L a crude operationalization of cognitive flexibility. Although that may be possible, the TMT-L has shown good concurrent validity with other measures of executive functions (Sánchez -Cubillo et al., 2009) as well as the Tower of London measured in the current sample. If anything, TMT-L likely underestimates measurement precision of this dynamic construct, cognitive flexibility, which could mean that effect sizes could actually be larger than what they appear in this study. Nevertheless, future research must continue to assess the nature of cognitive flexibility and develop more rigorous measures of it that increase precision beyond the TMT or Wisconsin card sort tasks.

Implications for Urban Mathematics Instruction

Altogether, these findings suggest urban math teachers would be wise to grasp the implications of real world applications, cognitive development, and race for mathematics learning in urban Black and Latino populations. Previous research (Gainsburg, 2008) as well as the current data reveal that TMAs are underutilized in urban mathematics instruction. On a seven-point scale, the 31 mathematics classrooms in this study averaged a score of 2.5 across hundreds of hours of focused observation in state-ranked as well as struggling schools alike. However, the present results intimate that TMAs should not be an ancillary luxury, only given when class time permits or when students inquire. Neither are TMAs only beneficial for students who have fully mastered the content. Such attitudes likely perpetuate inequity among the very students who have been historically underserved in mathematics (i.e., Black and Latino), limiting opportunities for them to see mathematics in a way that is congruent to what they value in their lives (Bartell, 2011). Although students who scored higher on cognitive flexibility were better able to process TMAs for their own attainment value, the main effect of TMAs on attainment value was still positive for the entire sample, indicating all students can benefit from TMAs. Considering the well-known declines in adolescent math value (Jacobs et al., 2002; McCoy, 2005), the positive prediction of TMAs on attainment value for students with lower cognitive flexibility should still be interpreted with optimism despite the nonsignificant slope. In light of pervasive value declines, even a flat slope could be considered encouraging.

When enacting TMAs, this data advises teachers to remember adolescent developmental factors that may prevent students from connecting with their applications initially. Due to the slow development of cognitive flexibility throughout adolescence (Diamond, 2013) as well as considering how cognitive load can further inhibit cognitive flexibility (Sweller et al., 1998), adolescents might be more likely to 'miss' the application message during early adolescence (i.e., Grades 5–9), when abstract mathematical concepts are being processed for the first time, or when the TMA is itself abstract or has multiple parts to it. However, many of these miscues and limitations in adolescent processing are natural, and teachers should not take this as feedback to discontinue their efforts in facilitating opportunities for TMAs.

As teachers consider how they should construct their TMAs, previous research has shown some basic guiding principles. First, the more the TMA is grounded in students' actual experiences, the less cognitive load it introduces into working memory (Goldstone & Son, 2005; Nathan, & Koedinger, 2000), making the math and application easier to process simultaneously. Thus, teachers should consider whether textbook examples or their own adult experiences with real-world mathematics may be too abstract to develop good examples for adolescents. Second and related, research has shown that story problems that revolved around home life, social interactions, social media, and family were more accessible (i.e., less abstract) to adolescents compared with story problems about the business world or physics concepts (Clinton, Walkington, & Howell, 2013). Last, as Black and Latino adolescents face additional challenges with processing negative cultural stereotypes and stigma, culturally responsive math examples (Gutstein, 2007; Turner et al., 2009) may help address these stereotypes, affirm cultural pride, and reduce cognitive load in processing the value of mathematics.

Considering issues of race and stigma, math teachers may also have the opportunity to learn from the alternative messengers in this study (e.g., parents of Shayla and Kasir), on the importance of socializing their adolescents to develop a critical consciousness of race, stigma, and mathematics. Although most urban math teachers may not see it as their job to elicit conversations about race, the current data and prior research (Walker, 2006, 2014) highlight its value for developing resilient identities that are grounded in pride, cultural legacy, and heritage. Therefore, the urgency is not only for connecting mathematics to the real world, but also engaging culturally responsive mathematics instruction to affirm racial identities and allow students to see connections between their racial identity and intellectual (i.e., mathematics) identity. Although Black and Latino adolescents may be aware of negative stereotypes through media and society, teachers and families can support them in critiquing and deconstructing such stereotypes versus ignoring them or fear of confirming them. Scholars in mathematics education are still studying the particulars of how to enact these initiatives (for a review see: Bartell, 2011).

Through the adolescent interviews, there is evidence to suggest that they struggled more with inferring present-tense math value versus future value. Thus, math teachers should be mindful about illustrating the value of mathematics for students' current lives, not just for future courses or careers. Alternative messengers, particularly parents and relatives, seemed adept at providing applications tailored to the current and individual interests of their adolescents. This may suggest powerful TMAs are an outgrowth of teachers getting to know their students well and developing quality relationships with them. Conducting math interest interviews in the beginning of the year and using this information to generate math examples throughout the year may be a good starting point (for more detail see Walkington, Sherman, & Howell, 2014).

Implications for Research

In addition to implications for teachers and families, there are also important implications for research in educational psychology. Although achievement values (e.g., attainment, utility) are well-established in educational psychology and acknowledged as reliable and valid through dozens of studies, our proclivity toward generalizability in quantitative research entices us to assume that these concepts are enacted similarly across most children. However, that assumption may be a careless one that undermines how culture and context provide informative nuance to these concepts and the diverse experiences of the people who enact them. This work also underscores the messiness of achievement values. Although the quantitative findings showed linear and predictable growth in attainment value, the qualitative findings revealed an intersection of multiple forms of achievement values, showing that an adolescent can possess utility, attainment, and abstract value simultaneously and at times tension between these values.

Future research on achievement values should consider new ways of operationalization that account for culture, context, or diverse ways these values may be expressed. Hence, person-centered (Snyder & Linnenbrink-Garcia, 2013) and other intersectional methods may become imperative in advancing knowledge on achievement values in educational psychology. Further, greater use of qualitative interviewing and naturalistic observation in combination with established questionnaire measures of achievement values are logical future steps.

Ultimately, this study highlights how psychological researchers can leverage the full potential of mixed methods for understanding achievement motivation among understudied populations. Using student voice in this study not only gave means for corroborating the quantitative findings but also unearthed racialized and contextualized notions of attainment value, which are not well understood in achievement motivation research (DeCuir-Gunby & Schutz, 2014). Thus, the present work becomes more than just a study using Black and Latino participants, rather it presents an opportunity to see how race, stigma, and context influence what mathematics means to these adolescents.

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(*Appendix follows*)

Appendix

Measures

Survey Items

Scale

Participants were asked the rate the following items on a 6-point scale
1 = Completely Disagree
2 = Mostly Disagree
3 = Disagree a Little
4 = Agree a Little
5 = Mostly Agree
6 = Completely Agree
Importance of Mathematics as Self-defining (i.e., attainment value of mathematics):

Being a very good math student is one of the most important things in my life

Being a very good math student is an important reflection of who I am

Being a very good math student defines who I am

I want to do well in math class to prove to myself that I am smart

Generalized Value of Mathematics:

I can understand why things that I am learning in math class are important

The things I am learning in math class are interesting

The things I am learning in math class are not very important overall (*Reverse coded*)

I like learning new things in math class

Interview Questions

Background

1. Tell me a little about yourself.
2. Tell me about your school experiences.
   a. Describe yourself as a student
      i. What would you say is your best subject in school? Why?
      ii. What is your favorite subject in school? Why?
      iii. Tell me about the grades you currently have in your classes
      iv. What do you think about your grades?

Transition

Interviewer states: "I am going to ask you some questions about one specific subject, math. I would like you to think about this year in math, starting from the first day of school in September. Think about your math classes, think about your teacher, and think about how you feel about math."

Importance

3. Tell me about an experience in your math class(es) from this past month.
   a. How do you feel about math?
   b. Can you give me an example? Can you tell me a story?
4. Tell me about a recent time this year you Aced a math test?
   a. Tell about a time this year you did poorly on a math test?
5. If a student gets a good grade in math, what do your friends do/say?
   a. Can you give me an example? Can you tell me a story?
6. Tell me a story about an experience in math class that was very memorable for you?
   a. How did this make you feel?
   b. How important is math to you?
7. You’ve explained how math is important to you personally. Now, how do you see math as being useful to know in other ways?

Learning Environment

8. Tell me about your math teacher.
9. Describe a recent experience you had with your math teacher.
   a. How did that make you feel?
   b. What does your teacher do to show you that math is important?
10. Describe a time that you were very frustrated in math class.
   a. What, when, why, how?
   b. What did your teacher do?

Family

11. Tell me a little about your family
   a. Who’s at home? Relationships? A normal day?
12. Tell me about the last time you had a conversation about school with one of your family members.
   a. What happened? When did it happen? Why did it happen?
   b. What did you learn from this conversation?
13. What stories does your family tell about their experiences at school?
14. What has your family told you about their experiences in math?

Racial and Ethnic Culture

15. Where does your family come from? What is your family’s culture?
16. What does it mean to you to be (use student term of identification here)?
17. What does your family tell you about your culture?

a. Describe an important story that your family told you about your culture.

b. Why was this story important?

c. How did it make you feel?

18. What do other people think about (use student's term of identification here)? Tell a story.

    a. How does that make you feel?

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